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FLOW VELOCITY CALCULATIONS FOR A "PERPETUAL SALT FOUNTAIN"

KENDALL GODDARD HINMAN, JR.

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FLOW VELOCITY CALCULATIONS FOR A "PERPETUAL SALT FOUNTAIN"

by

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Submitted in partial fulfillment
for the degree of

MASTER OF SCIENCE

from the

UNITED STATES NAVAL POSTGRADUATE SCHOOL

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ABSTRACT

The "perpetual salt fountain" can occur in an area of the ocean where the water is vertically stable. Warm, saline water overlies cold, relatively fresh water. A tube is placed in the water from just beneath the ocean surface to just beneath the halocline. Water is pumped up the tube to initiate the flow. As the water in the tube is warmed, it becomes less dense than (and thus buoyant with respect to) the surrounding water, and the flow will continue after pumping is stopped.

The effect of the vertical conduction of heat by turbulent mixing on the velocity of flow in the "perpetual salt fountain" is investigated. Temperature and salinity profiles with depth from hydrographic soundings are considered to determine flow velocities in the ocean.

The calculations show that vertical heat conduction has a negligible effect on the rate of flow in the presence of horizontal heat conduction, the value of the average salinity of a column of water minus the salinity of the water entering the bottom of the tube is the primary factor influencing the rate of flow, and the effect of the temperature-depth profile on the velocity of flow is negligible so long as initial stability requirements are met.

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LIST OF SYMBOLS

w	wall thickness of tube
ρ	density of sea water
c	specific heat of sea water
ν	kinematic viscosity of sea water
g	acceleration due to gravity
R	Reynolds number
τ	stress on inner surface of the tube
K	coefficient of heat conduction for turbulent flow
v	average velocity of flow in the tube
n	heat conductivity of tube material
r	inner radius of tube
y	depth below the ocean surface
T	temperature
p	pressure
h	depth of bottom of the tube
L	length of the tube
$\theta(y)$	temperature of water outside the tube at depth y
$\phi(y)$	temperature of water inside the tube at depth y
$S(y)$	salinity of water outside the tube at depth y
$S(h)$	salinity of water at depth h, and salinity of water inside the tube

$\gamma = 2nL/wcrvh\rho$

$\alpha = 2n/Krw$

$\beta = hcv\rho/LK$

$a = KL/cvh\rho$

$$\lambda_1 = \frac{-\beta - (\beta^2 + 4\alpha)^{1/2}}{2}$$

$$\lambda_2 = \frac{-\beta + (\beta^2 + 4\alpha)^{1/2}}{2}$$

$$D = \frac{1}{\alpha^4}\left[24C\alpha^2 + 72\beta^2 C\alpha + 24\beta^4 C\right]$$

$$E = \frac{1}{\alpha^3}\left[48\beta C\alpha + 24\beta^3 C\right]$$

$$F = \frac{1}{\alpha^2}\left[12C\alpha + 12\beta^2 C\right]$$

$$G = \frac{1}{\alpha}\left[4\beta C\right]$$

$$\bar{S} = \frac{1}{h}\int_0^h S(y)\,dy$$

1. Introduction.

Stommel, Arons, and Blanchard [8] first wrote about an "oceanographic curiosity" which they called the "perpetual salt fountain." Consider a situation in which there is a layer of cold, relatively fresh water which underlies a warm, saline upper layer of water. The density of water is a function of temperature, salinity and pressure, and the density of the upper layer must be less than the density of the lower layer for stability.

Now insert a tube into the water which extends from just beneath the surface of the water to the top of the lower layer of water. A power source is provided to initiate an upward flow of water. As the water in the lower layer is started up the tube, it is warmed by heat flowing into the tube from the surrounding warm water. Since the water in the tube is less saline than the water outside the tube, it becomes less dense than the outside water and is forced upward.

The flow process requires no source of power other than that used to initiate the flow, and flow will continue so long as there continues to be a well-defined two-layer system. The principle will work for a flow in either direction.

Groves [2] calculated the rate of this flow in the tube to determine what, if any, practical value this perpetual flow phenomenon might have. Groves' calculation was based on a simple model where the water temperature was considered a constant down to the bottom of the tube which just extended into water of a colder temperature. Groves determined that the volume rate of flow for his tube would be too small to be of practical significance for fish farming (that is, use of this principle as a source for providing nutrient-rich deep water at the surface to increase the amount of edible fish in an area).

Groves' model neglected the effects of vertical heat conduction within the tube, which should increase the warming of the water inside the tube and provide a corresponding increase in rate of flow. If this were the case, then perhaps a larger tube could be used.

The additional heating which is a result of vertical heat conduction would be sufficient to warm the increased volume of water in a larger tube. If the flow velocity in a tube of larger inner radius was approximately equal to the flow velocity in the tube of smaller inner radius, the tube with the larger inner radius would provide an increase in volume rate of flow.

It was considered that a significant increase in the velocity of the water in the tube might increase the volume of flow sufficiently to warrant further investigation of this process and its practical applications. The effects of wall friction on the temperature of the water inside the tube were considered, but the result was so small that the effect was considered negligible. Calculations of temperature change due to this stress are contained in Appendix C.

2. Solution for a model which includes the effects of vertical heat conduction.

In order to see the effect of vertical heat conduction on the rate of flow of the water inside the tube, let us consider the same model that Groves worked with, but include vertical heat conduction.

Assume that the water density is a linear function of temperature and salinity, and that the water is incompressible. Therefore, the rate of change of density with temperature, $\partial\rho/\partial\theta$, and the rate of change of density with salinity, $\partial\rho/\partial S$, are constant, a valid assumption within the small range of temperature and salinity considered. Then,

$$\rho = \rho_0 + \theta\left(\partial\rho/\partial\theta\right)_0 + S\left(\partial\rho/\partial S\right)_0 \tag{1}$$

where ρ is the water density and Θ and S respectively are the temperature and salinity of the water. The heat flow into the tube is calculated by the equation

$$\frac{Dq}{Dt}\left(\rho\pi r^2 dy\right) = n\, 2\pi r\, dy \frac{(\Theta-\Phi)}{w} + K\pi r^2 dy \frac{d^2\Phi}{dy^2} \tag{2}$$

where D/Dt designates the material derivative following the mean motion of the water, q is the heat per unit mass, n is the heat conductivity of tube material (say, copper), r is the inner radius of the tube, Ø(y) is the temperature of the water in the tube, w is the wall thickness of the tube, and K is the coefficient of vertical heat conduction for turbulent flow calculated in Appendix D. At the sea surface y=0, and y is positive downward. If we assume a steady state,

$$\frac{D\Phi}{Dt} = \frac{\partial\Phi}{\partial y}\frac{\partial y}{\partial t} = -v\frac{\partial\Phi}{\partial y} \tag{3}$$

since the local change $\partial\Phi/\partial t = 0$. (The average flow velocity is v.) Combining the constant terms, equation (2) becomes

$$\frac{d^2\Phi}{dy^2} + \beta\frac{d\Phi}{dy} - \alpha\Phi = -\alpha\Theta \tag{4}$$

where β=hcvρ/LK and α=2n/Krw. The specific heat of sea water is c, h is the depth of the bottom of the tube, and L is the length of the tube.

The general solution of the homogeneous equation associated with (4) is

$$\Phi_G(y) = J_1 e^{\lambda_1 y} + J_2 e^{\lambda_2 y} \tag{5}$$

where J_1 and J_2 are undetermined constant coefficients,

$$\lambda_1 = \frac{-\beta-(\beta^2+4\alpha)^{1/2}}{2} \tag{6}$$

and

$$\lambda_2 = \frac{-\beta+(\beta^2+4\alpha)^{1/2}}{2} \tag{7}$$

.

2.1. Solutions for fourth-power temperature and salinity profiles.

In order to find a particular solution to equation (4), a fourth-power temperature profile was assumed:

$$\Theta(y) = A - Cy^4 . \tag{8}$$

This function was chosen because it can be easily fit to actual temperature-depth curves. The value of sea-surface temperature can be inserted for A, and the value $(T_s - T_h)/h^4$ inserted for C. The complete solution for equation (4) is

$$\begin{aligned}\Phi(y) = {} & J_1 e^{\lambda_1 y} + J_2 e^{\lambda_2 y} + A - \frac{1}{\alpha^4}\left[24C\alpha^2 + 72\beta^2 C\alpha + 24\beta^4 C\right] \\ & - \frac{1}{\alpha^3}\left[48\beta C\alpha + 24\beta^3 C\right]y - \frac{1}{\alpha^2}\left[12C\alpha + 12\beta^2 C\right]y^2 \\ & - \frac{1}{\alpha}\left[4\beta C\right]y^3 - Cy^4 .\end{aligned} \tag{9}$$

The water entering the bottom of the tube is at the same temperature as the surrounding water at that depth; therefore, $\phi(h)=\theta(h)$. For steady flow, the water at the top of the tube is assumed to be at the same temperature as the water surrounding it. This is a reasonable assumption since we are considering vertical heat conduction, and the water at the top of the tube undergoes vertical turbulent mixing with the surrounding water. Thus, the other boundary condition is $\phi(0)=\theta(0)$. Only upward flow is considered in this paper.

Applying the boundary conditions to equation (9) and solving for J_1 and J_2,

$$\begin{aligned}\Phi(y) = {} & \left[\frac{D(1-e^{\lambda_2 h}) + Eh + Fh^2 + Gh^3}{e^{\lambda_1 h} - e^{\lambda_2 h}}\right] e^{\lambda_1 y} \\ & + \left[\frac{D(e^{\lambda_1 h}-1) - Eh - Fh^2 - Gh^3}{e^{\lambda_1 h} - e^{\lambda_2 h}}\right] e^{\lambda_2 y} \\ & + A - D - Ey - Fy^2 - Gy^3 - Cy^4 .\end{aligned} \tag{10}$$

The constants D, E, F, and G are functions of α, β , and C for a given velocity:

$$D = \frac{1}{\alpha^4}\left[24C\alpha^2 + 72\beta^2 C\alpha + 24\beta^4 C\right],$$

$$E = \frac{1}{\alpha^3}\left[48\beta C\alpha + 24\beta^3 C\right],$$

$$F = \frac{1}{\alpha^2}\left[12C\alpha + 12\beta^2 C\right],$$

$$G = \frac{1}{\alpha}\left[4\beta C\right].$$

Now assume that the water is removed from the top of the tube as fast as it flows up so that no additional pressure head is formed. Then the pressure difference driving the flow is the hydrostatic pressure outside the tube at y=h, less the hydrostatic pressure in the tube at this depth.

Equation (1) gives an internal pressure difference of

$$p_i(h) - p_i(0) = \int_0^h dp_i = g\int_0^h \left[\rho_0 + \left(\frac{\partial\rho}{\partial S}\right)_0 S(h) + \left(\frac{\partial\rho}{\partial\theta}\right)_0 \Phi(y)\right] dy \tag{11}$$

and an external pressure difference of

$$p_o(h) - p_o(0) = g\int_0^h \left[\rho_0 + \left(\frac{\partial\rho}{\partial S}\right)_0 S(y) + \left(\frac{\partial\rho}{\partial\theta}\right)_0 \Theta(y)\right] dy\ . \tag{12}$$

Since both pressures are atmospheric at the surface, subtraction of equation (12) from equation (11) gives the pressure which drives the flow:

$$\Delta p = g\int_0^h \left\{\left[S(y) - S(h)\right]\frac{\partial\rho}{\partial S} + \left[\Theta(y) - \Phi(y)\right]\frac{\partial\rho}{\partial\theta}\right\} dy\ . \tag{13}$$

Density is constant to within one per cent, and from the continuity equation, ρ v=constant. This implies that v$\doteq$constant and, in fact, will be assumed constant. Then the wall friction, τ, is also constant:

$$\tau = \frac{1}{8} f(R)\,\rho v^2. \tag{14}$$

Here, f(R) is called the "resistance coefficient," (Rouse [5]). Due to the small value of viscosity for sea water, the flow will be assumed to be turbulent, and the relationship

$$f(R) = .3164/R^{1/4} \tag{15}$$

for smooth tubes will be used. Here, R is the Reynolds Number,

$$R = 2vr/\nu \tag{16}$$

where ν is the kinematic viscosity of sea water.

Since the water in the tube is not accelerating, the driving force due to pressure must equal the retarding force due to wall friction:

$$\Delta p = \frac{2L\tau}{r}\,. \tag{17}$$

Combining equations (14) and (17) gives

$$\frac{v^2 \rho L f(R)}{4r} - \Delta p = 0. \tag{18}$$

Solving explicitly for Δp in equation (13) necessitates prescribing a salinity-depth curve. A fourth-power salinity profile

$$S(y) = Q - Py^4 \tag{19}$$

gives the salinity curve with depth the same physically reasonable form as the temperature curve. The surface salinity is Q and $P=(S_S-S_h)/h^4$. Equation (13) becomes

$$\Delta p = gh\frac{\partial\rho}{\partial S}\left[Q - \frac{Ph^4}{5} - S(h)\right] - g\frac{\partial\rho}{\partial\theta}\left[\frac{(1-e^{\lambda_1 h})(1-e^{\lambda_2 h})\{D(\lambda_1-\lambda_2)\}}{\lambda_1\lambda_2(e^{\lambda_1 h}-e^{\lambda_2 h})}\right] \tag{20}$$

$$- g\frac{\partial\rho}{\partial\theta}\left[\frac{(Eh+Fh^2+Gh^3)\{\lambda_1(1-e^{\lambda_2 h})-\lambda_2(1-e^{\lambda_1 h})\}}{\lambda_1\lambda_2(e^{\lambda_1 h}-e^{\lambda_2 h})}\right].$$

Equation (20) is difficult to evaluate in its present form, but there are several approximations which can be made to simplify the expression without sacrificing accuracy.

First, assume that the rate of flow will be less than ten meters per second. Calculated values were much less than this value, and a velocity of ten meters per second would require an extremely large pressure force to drive the flow.

Second, assume that the tube thickness is very much less than the inner radius of the tube. A thin tube is required to allow the water in the tube to reach approximately the same temperature as the water outside the tube as it flows upward.

Third, assume terms that are 10^{-6} (order of magnitude) as large as the dominant terms are negligible.

From equation (5), λ_1, is a function of α and β and will always be a negative number. If h is greater than (say) 50 meters, (it will usually be much greater than this), $\lambda_1 h$ is a large negative number, and $e^{\lambda_1 h}$ is negligible.

The constants D, E, F, and G are such small numbers that, even when multiplied by 1, h, h^2, and h^3 respectively, they are still negligible compared to other terms.

Equation (20) becomes

$$\Delta p \doteq g h \frac{\partial \rho}{\partial S}\left[Q - \frac{P h^4}{5} - S(h)\right] \tag{21}$$

and combining equations (18) and (21) gives

$$v^{7/4}\left[6.65 \times 10^{-2}\, \frac{\rho L}{r}\left(\frac{\nu}{r}\right)^{1/4}\right] - g h \frac{\partial \rho}{\partial S}\left[Q - \frac{P h^4}{5} - S(h)\right] = 0. \tag{22}$$

This is a closed form of the solution for the flow velocity which includes the effects of vertical heat conduction.

If vertical heat conduction is not considered, the governing differential equation (2) simplifies to

$$\frac{Dq}{Dt}\left(\rho \pi r^2 dy\right) = m\, 2\pi r\, dy \frac{(\Theta - \Phi)}{W} \tag{23}$$

which has the solution

$$\Phi(y) = \gamma e^{\gamma y}\int_y^h e^{-\gamma T}\,\Theta(T)\,dT + \Theta(h)\,e^{-\gamma(h-y)} \tag{24}$$

where $\gamma = 2nL/wcvrh\rho$.

Solving equation (24) for the fourth-power temperature profile gives

$$\begin{aligned}\Phi(y) = {} & A - Cy^4 - \frac{1}{\gamma}4Cy^3 - \frac{1}{\gamma^2}12Cy^2 - \frac{1}{\gamma^4}24C(\gamma y + 1) \\ & + e^{-\gamma(h-y)}\left[\frac{4Ch^3}{\gamma} + \frac{12Ch^2}{\gamma^2} + \frac{24C(\gamma h+1)}{\gamma^4}\right].\end{aligned} \tag{25}$$

Using the fourth-power salinity profile and equation (25) for $\phi(y)$, equation (13) becomes

$$\Delta p \doteq gh\frac{\partial\rho}{\partial S}\left[Q - \frac{Ph^4}{5} - S(h)\right] - g\frac{\partial\rho}{\partial\Theta}\left[\frac{Ch^4}{\gamma} + \frac{24C}{\gamma^5}\right] \tag{26}$$

where negligibly small terms have been ignored. The solution for fourth-power temperature and salinity profiles when vertical heat conduction is not considered is

$$r^{7/4}\left[6.65\times 10^{-2}\frac{\rho L}{r}\left(\frac{\nu}{r}\right)^{1/4}\right] - gh\frac{\partial\rho}{\partial S}\left[Q - \frac{Ph^4}{5} - S(h)\right] - g\frac{\partial\rho}{\partial\Theta}\left[\frac{Ch^4}{\gamma} + \frac{24C}{\gamma^5}\right] = 0. \tag{27}$$

Equations (22) and (27) were solved using the temperature and salinity profiles pictured in figure 1, using identical tube dimensions. The flow velocity which was influenced by vertical heat conduction was less than one-tenth of one per cent greater than the flow velocity calculated ignoring vertical heat conduction (Table 1.).

2.2 Solutions for linear temperature and salinity profiles.

In order to consider the effect of different temperature and salinity profiles, linear profiles were assumed:

$$\Theta(y) = M - Ny \tag{28}$$

and

$$S(y) = Q - Py\,. \tag{29}$$

The solution to equation (4) then becomes

$$\Phi(y) = X\left[\frac{e^{\lambda_2 h} - 1}{e^{\lambda_2 h} - e^{\lambda_1 h}}\right]e^{\lambda_1 y} + X\left[\frac{1 - e^{\lambda_1 h}}{e^{\lambda_2 h} - e^{\lambda_1 h}}\right]e^{\lambda_2 y} + M - Ny - X \tag{30}$$

where $X = N\beta/\alpha$.

Solving equation (13) using the temperature profile of equation (30) gives

$$\Delta p \doteq gh\frac{\partial\rho}{\partial S}\left[Q - \frac{Ph}{2} - S(h)\right] + gh\frac{\partial\rho}{\partial\Theta}X + g\frac{\partial\rho}{\partial\Theta}X\frac{(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2} \tag{31}$$

where negligibly small terms have been ignored. The solution for linear temperature and salinity profiles, including vertical heat conduction effects is

$$\begin{aligned} &v^{7/4}\left[6.65\times10^{-2}\frac{\rho L}{r}\left(\frac{\nu}{r}\right)^{1/4}\right] - gh\frac{\partial\rho}{\partial S}\left[Q - \frac{Ph}{2} - S(h)\right] - gh\frac{\partial\rho}{\partial\Theta}X \\ &\quad - g\frac{\partial\rho}{\partial\Theta}X\frac{(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2} = 0. \end{aligned} \tag{32}$$

Assuming linear temperature and salinity profiles, equation (24) becomes

$$\Phi(y) = \frac{N}{\gamma}\left[e^{-\gamma(h-y)} - 1\right] + M - Ny \tag{33}$$

and the solution to equation (13) is

$$\Delta p \doteq gh\frac{\partial\rho}{\partial S}\left[Q - \frac{Ph}{2} - S(h)\right] - g\frac{\partial\rho}{\partial\Theta}\left[\frac{N}{\gamma^2} - \frac{N}{\gamma}\right] \tag{34}$$

where negligibly small terms have been ignored. Thus, the solution for flow velocity for linear temperature and salinity profiles where vertical heat conduction is ignored is

$$V^{7/4}\left[6.65\times10^{-2}\,\frac{PL}{r}\left(\frac{\nu}{r}\right)^{1/4}\right]-gh\frac{\partial\rho}{\partial S}\left[Q-\frac{Ph}{2}-S(h)\right]+g\frac{\partial\rho}{\partial\theta}\left[\frac{N}{\gamma^2}-\frac{N}{\gamma}\right]=0. \tag{35}$$

Equations (32) and (35) were solved using the temperature and salinity profiles pictured in Figure 2, with identical tube dimensions. Again, the rate of flow influenced by vertical heat conduction was less than one-tenth of one percent greater than the rate of flow calculated ignoring vertical heat conduction. (Table 1.).

It is interesting to compare the actual calculations of flow velocity for the profiles in Figure 1 with those related to Figure 2. Vertical heat conduction was included, but this is not pertinent to the discussion below. The velocity solutions, equations (22), (25), (32) and (34) all contain the term $\left[Q-Ph/2-S(h)\right]$ which is the average salinity of the water between the surface and the depth y=h, $\overline{S}$, less the salinity at y=h, S(h). In making these calculations it was noted that the main factor causing the one centimeter per second difference in rate of flow between the profiles in Figures 1 and 2 was the value of the $\overline{S}$-S(h) term.

Calculating the value of $\overline{S}$-S(h) for the salinity profiles in Figure 1 and in Figure 2 showed that the two values were almost identical. However, the value of $\overline{S}$ obtained by applying the fourth-power salinity profile approximation to the salinity curve of Figure 1 gave an erroneous value for $\overline{S}$-S(h). The error in the calculation of $\overline{S}$ by substituting an approximate function for the actual salinity curve caused a marked error in flow velocity result. This error was later verified by the computer program determination of flow velocity which required no approximation of $\overline{S}$. This indicates the great importance of the salinity profile which will be discussed in more detail below.

The flow rates for the θ-S profiles contained in Figures 1 and 2 were also calculated utilizing a digital computer (see below). Here the effects of vertical heat conduction were not included. The rate of flow in each case was within one per cent of the other, and was essentially the same as the flow rate calculated by hand from equation (32).

TABLE 1

RESULTS OF CALCULATIONS FOR RATE OF FLOW

Case No.	θ(y) & S(y) Profiles	"Fixed # Parameters"	Hand ## Calculations	Computer ## Calculations
1	Figure 1	Approximate		17.32
2	Figure 2	Approximate	17.36	17.30
3	Figure 1	Approximate	16.32*	
4	Figure 2	Approximate**	28.52	27.88
5	Figure 1	Precise		17.40
6	Figure 3	Precise		22.68
7	Figure 4	Precise		26.60
8	Figure 5	Precise		20.89
9	Figure 6	Precise		8.03
10	Figure 5	Precise***		30.94
11	Figure 3	Precise***		33.59

* The flow velocity result is for the hand calculation of the fourth-power approximation of temperature and salinity profiles illustrated in Figure 1.

** The inner radius of the tube was increased to 20 centimeters and the thickness of the tube increased to two centimeters.

*** The tube was inserted vertically so that the length of the tube equals the depth of the bottom of the tube.

# Appendix E contains "fixed parameter" values. Except where otherwise noted, the inner radius of the tube is 0.1 meters and the wall thickness is 0.002 meters.

## Flow velocity results are in centimeters per second.

Marked differences in rate of flow which occur when the tube is inserted into water which has different temperature and salinity profiles are a direct result of the difference in the salinity profile from one location to another. The temperature profiles did not significantly influence the rate of flow within the small but realistic range of temperature and salinity used by the author.

2.3 Effects of vertical heat conduction.

The importance of vertical heat conduction was investigated in another manner. The wall thickness of the tube was increased by a factor of ten. The flow velocity decreased, but the decrease was less than one per cent. Increasing the inner radius of the tube from ten centimeters to 20 centimeters and retaining the thicker-walled tube resulted in a significant increase in rate of flow of 37%.

Comparison of the results of the above calculations indicated that the vertical heat conduction due to vertical turbulent mixing was sufficient to warm the water in the tube. In order to determine the extent of the warming due to vertical heat conduction, the velocity of flow was then determined for the situation in which no horizontal heat conduction was allowed. In this case, the temperature in the tube is a function only of vertical heat conduction. The governing differential equation (2) simplifies to

$$\frac{Dq}{Dt}\rho\pi r^2 dy = K\pi r^2 dy \frac{\partial^2\Phi}{\partial y^2} \qquad (36)$$

which has the solution

$$\Phi(y) = \left[\frac{\Theta(0)-\Theta(h)}{1-e^{-ah}}\right]e^{-ay} + \Theta(0) - \left[\frac{\Theta(0)-\Theta(h)}{1-e^{-ah}}\right] \qquad (37)$$

where $a = KL/cvh\rho$.

Solving equation (13) using equation (26) and linear temperature and salinity profiles,

$$v^{7/4}\left[6.65\times10^{-2}\,\frac{\rho L}{r}\left(\frac{\nu}{r}\right)^{1/4}\right]-gh\frac{\partial\rho}{\partial S}\left[Q-\frac{Ph}{2}-S(h)\right]+g\frac{\partial\rho}{\partial\theta}\left[\frac{Nh^2}{2}+\frac{Nh}{\alpha}\right]=0 \quad (38)$$

where negligibly small terms have been omitted.

The resulting flow velocity is nearly identical to the flow velocity calculated using the same tube, but allowing horizontal heat conduction. The process of heat conduction in the vertical by turbulent mixing would require a longer period of initial pumping to reach a steady state condition, but the results indicate that the vertical heat conduction process is sufficient to heat the water inside the tube.

The initial tube placement was at an angle of depression of 30 degrees to the water surface. This was done so that the water in the tube would require a longer time interval to reach the surface and undergo increased warming. Since the water in the tube is sufficiently warmed by vertical heat conduction to be less dense than the water surrounding it outside the tube, the tube placement was changed to a vertical position. The decrease in tube length resulted in a corresponding decrease in the effect of wall friction and a 48% increase in rate of flow. Such an increase is certainly significant and will increase the volume rate of flow.

3. Computer solutions.

In order to determine the rate of flow for a "perpetual salt fountain" situated in the ocean, actual oceanographic temperature and salinity data for a given area must be known. A program to obtain flow velocity solutions which neglected vertical heat conduction was written for a CDC 1604 digital computer using FORTRAN 60 programming. Using this program, flow velocity was easily determined for any area in which temperature and salinity data were available.

Temperature and salinity values for each one-meter interval over the entire length of the tube, tube dimensions, and "fixed parameters" are entered on data cards in the program. "Fixed parameters" include kinematic viscosity of sea water (ν), density of sea water (ρ), specific heat of sea water (c), acceleration of gravity (g), rate of change in density with change in salinity ($\partial\rho/\partial S$) and rate of change in density with change in temperature ($\partial\rho/\partial\theta$). Appendix E contains the values used for "fixed parameters."

Subroutine INTE computes the temperature profile in the tube, and subroutine DELTP computes the pressure force which drives the flow. Subroutine INTE provided some interesting problems due to the size of the exponents of the exponential functions. A discussion of these problems, and the solution to them is included with the computer program in Appendix A. Subroutine DELTP is a trapezoidal integration which is as accurate as can be obtained from the small number of data points generally given in a hydrographic sounding.

Initial data used in the program was set up to treat the same case as was calculated by Groves. Temperature and salinity profiles were devised so that values of $\theta(0)-\theta(h)$, $\overline{S}-S(h)$, and the "fixed parameters" would be identical to those used by Groves. Additional test cases were run to compare with the results of hand calculations.

Values for the constant terms $\partial\rho/\partial\theta$ and $\partial\rho/\partial S$ were then calculated from the equation of state given by Eckart [1] (see Appendix B) and more precise values for the other "fixed parameters" were obtained and used with data from actual hydrographic soundings. Several cases of actual data were employed to get an idea of the range of values of flow velocity which could be expected in the ocean. The results are contained in Table 1, and the temperature-depth and salinity-depth curves are pictured in Figures 1-6.

In the actual ocean data cases, the depth to which the tube was extended was determined by the depth of the halocline. The tube was always extended into the water just below a negative halocline.

4. Results.

4.1 Vertical heat conduction versus no vertical heat conduction.

Initial flow velocity calculations were made by hand using the initial values of the "fixed parameters" contained in Appendix E. The rate of flow for identical tube dimensions was calculated for θ-S profiles contained in Figures 1 and 2.

The calculation of flow rate was made twice for the linear θ-S profiles of Figure 2, once including the effects of vertical heat conduction and once neglecting the effects of vertical heat conduction. In each case, inclusion of vertical heat conduction had a negligible effect on the rate of flow.

Flow velocity calculations were then made for the θ-S profiles of Figure 1 using the fourth-power temperature and salinity profiles. These functions were selected because they approximated the physical appearance of the curves in Figure 1 and because they were easily integrable. Flow rate solutions for the vertical heat conduction and no vertical heat conduction equations were again identical to four significant figures.

Flow velocity was then calculated for a situation in which no horizontal conduction of heat was allowed, so that only vertical heat conduction was possible. The rate of flow for this calculation was nearly identical to the rate of flow calculated for two other cases which used the same tube dimensions, "fixed parameter" values, and θ-S profiles. The first allowed both horizontal and vertical heat conduction, and the second allowed only horizontal heat conduction.

On the basis of these calculations, it was concluded that the effect of vertical heat conduction on the rate of flow of the water inside the tube was negligible in the presence of horizontal conduction. Thus it was decided to make use of the computer program which did not include the effects of vertical heat conduction for the remaining calculations.

4.2 Computer results versus Groves' result.

Utilizing the computer program allowed exact values of temperature and salinity at each depth to be used. Temperature and salinity values from Figures 1 and 2 were used in the program with a resulting rate of flow of 17.30 centimeters per second (cps) for Figure 1 profiles and 17.32 cps for Figure 2 profiles. The 17.32 cps flow velocity associated with the profiles of Figure 2 was in close agreement with the rate of flow calculated by hand for the same profiles. The 17.30 cps rate of flow calculated for the profiles in Figure 1 was much greater than the rate of flow calculated by hand for the same profiles. Investigation of the hand calculation revealed that the $\overline{S}-S(h)$ term calculated by hand differed significantly from the actual value of $\overline{S}-S(h)$ obtained from Figure 1. This difference was attributed to use of the fourth-power salinity profile as an approximation of the actual salinity profile when making hand calculations. $\overline{S}-S(h)$ calculated from the curve of the fourth-power salinity profile is significantly different from the true value of $\overline{S}-S(h)$ in Figure 1. The influence of the approximation to the form of the temperature profile, however, was determined to be negligible.

The profiles of temperature and salinity in Figures 1 and 2 were constructed so that the values of $\overline{S}-S(h)$ and $\theta(0)-\theta(h)$ would be identical to those selected by Groves. Tube dimensions and "fixed parameter" values were also those used by Groves. Groves' calculated rate of flow was 17.50 cps as compared with a rate of flow of 17.32 cps obtained from the computer program. The difference in flow velocities can be attributed to a rounding-off procedure used by Groves to simplify calculations.

The values of $\partial\rho/\partial\theta$ and $\partial\rho/\partial S$ calculated in Appendix B were then inserted into the original program along with more precise values of the other "fixed parameters" to determine what effects the more precise values would have on the rate of flow. The resulting velocity increase of eight centimeters per second was not significantly greater, but the more precise values for the "fixed parameters" were retained for the remaining temperature and salinity profiles.

Temperature and salinity data from actual hydrographic soundings were also used. Figures 3 through 6 illustrate the actual profiles used. It was verified that areas of most pronounced negative haloclines produced the greatest rates of flow.

A marked increase in rate of flow was noted when the tube was inserted vertically instead of at a depression angle of 30 degrees to the ocean surface. The 48% increase in speed of flow will provide an equal increase in volume rate of flow.

Rate of flow was also increased by increasing the inner radius of the tube. This is doubly significant since an increase in flow velocity increases volume rate of flow, and increased tube radius will also increase the volume rate of flow.

5. Conclusions and acknowledgements.

The test cases discussed in this paper have demonstrated some interesting features. First, the temperature-depth structure is of relatively little importance. The surface layer must be sufficiently warm to insure stability, but the magnitude of the temperature change between the surface and the bottom of the tube has a negligible effect on the velocity of flow.

Second, the primary parameter influencing the rate of flow of water in the tube is the difference between the average salinity in the layer through which the tube extends, and the salinity of the water entering the bottom of the tube. To calculate the rate of flow for a given case to within one per cent of its precise value, a salinity-depth curve must be known. A simple area-averaging process can be used to determine $\overline{S}$, the average salinity, and S(h) is known. Then,

$$v^{7/4}\left[6.65\times10^{-2}\,\frac{\rho L}{r}\left(\frac{\nu}{r}\right)^{1/4}\right] \doteq g\,h\,\partial\rho/\partial S\left[\overline{S}-S(h)\right]. \tag{39}$$

Third, equation (39) gives excellent results because the effects of vertical heat conduction heat the water in the tube sufficiently to make it buoyant. Although vertical heat conduction does not directly increase the velocity of flow, the warming effects allow the substitution of a poorer heat-conducting material for the copper tube. A metal which is resistant to sea water corrosion would be a better material for the tube. In addition, a relatively thick tube with a larger inner radius may be used. This will increase the volume rate of flow and provide a more durable tube.

For example, using data from the equatorial Atlantic Ocean region in case #11 (Figure 3), the volume rate of flow is 10.5 liters per second. This is a 90% increase in volume rate of flow over that calculated by Groves. When the inner radius of the tube was enlarged in case #4 (Figure 2), the volume rate of flow increased to 34.6 liters per second. This is an increase of 530%, which is definitely significant.

It is therefore concluded that the "perpetual salt fountain" is more than an "oceanographic curiosity." Upward flow of nutrient-rich water from well below the surface in sufficient volume could make a fertile fishing area out of an area which is, at present, barren. A relatively inexpensive and simple method for providing this nutrient-rich water, such as the method described in this paper, is worthy of additional experimentation.

The author gratefully acknowledges the guidance, encouraging assistance and critical comments of Assistant Professor T. Green. My thanks also go to Mr. R. D. Brunell of the United States Naval Postgraduate School's computer facility staff for his valuable programming assistance.

APPENDIX A

COMPUTER PROGRAM FOR FLOW VELOCITY CALCULATIONS

```
..JOB0532FM,HINMAN,K.G.,FLOWUP
      PROGRAM FLOWUP
      DIMENSION T(401), S(401), PHI(401)
      COMMON NVT,H,GAMMA,DELH,PHI,S,T,SALTH,THETH,G,DELTP,CHDS,CHDT
      READ 50, NVT
   50 FORMAT (12I5)
      READ 10, (T(I),S(I),I=1,NVT)
   10 FORMAT (6E12.6)
      READ 10, R,E,CON,PL,W,D,SH,DEPTH,V0,EP1,THETH
      READ 10, H,G,DELH,SALTH,CHDS,CHDT
      PRINT 12, R,E,CON,PL,W,D
      PRINT 12, SH,DEPTH,V0,EP1,THETH
      PRINT 12, H,G,DELH,SALTH,CHDS,CHDT
   12 FORMAT (6E12.6)
      V=V
   19 V=V+0.01
      RE=2.0*V*R/E
      GAMMA=4.0*CON*PL/(W*D*SH*E*RE*DEPTH)
      CALL INTE
      CALL FINAL
      IF(DELTP-0.0) 55,55,56
   55 V=V-0.01
      GO TO 57
   56 TAU=DELTP*R/(2.0*PL)
      IF(RE-2000.0) 20,20,21
   20 FUNC=64.0/RE
      GO TO 22
   21 FUNC=0.3164/((RE)**(1.0/4.0))
   22 VEL=(8.0*TAU/(FUNC*D))**(1.0/2.0)
   11 FORMAT (30H  THE UPWARD FLOW VELOCITY IS 2E20.6)
      IF(VEL-V) 55,24,19
   57 V=V+0.001
      RE=2.0*V*R/E
      GAMMA=4.0*CON*PL/(W*D*SH*E*RE*DEPTH)
      CALL INTE
      CALL FINAL
      IF(DELTP-0.0) 40,40,41
   40 V=V-0.001
      GO TO 42
   41 TAU=DELTP*R/(2.0*PL)
      IF(RE-2000.0) 43,43,44
   43 FUNC=64.0/RE
      GO TO 45
   44 FUNC=0.3164/((RE)**(0.25))
   45 VEL=(8.0*TAU/(FUNC*D))**(0.5)
      IF(VEL-V) 40,24,57
   42 V=V+0.0001
```

```
      RE=2.0*V*R/E
      GAMMA=4.0*CON*PL/(W*D*SH*E*RE*DEPTH)
      CALL INTE
      CALL FINAL
      IF(DELTP-0.0) 46,46,47
   46 V=V-0.0001
      GO TO 70
   47 TAU=DELTP*R/(2.0*PL)
      IF(RE-2000.0) 71,71,72
   71 FUNC=64.0/RE
      GO TO 73
   72 FUNC=0.3164/((RE)**(1.0/4.0))
   73 VEL=(8.0*TAU/(FUNC*D))**(1.0/2.0)
      IF(VEL-V) 46,24,42
   70 V=V+0.00001
      RE=2.0*V*R/E
      PRINT 60, RE
   60 FORMAT (25H  THE REYNOLDS NUMBER IS E20.6)
      GAMMA=4.0*CON*PL/(W*D*SH*E*RE*DEPTH)
      PRINT 61, GAMMA
   61 FORMAT (14H GAMMA EQUALS E20.6)
      CALL INTE
      CALL FINAL
      PRINT 63, DELTP
   63 FORMAT (14H DELTP EQUALS E20.6)
      TAU=DELTP*R/(2.0*PL)
      PRINT 62, TAU
   62 FORMAT (12H TAU EQUALS E20.6)
      IF(RE-2000.0) 74,74,75
   74 FUNC=64.0/RE
      GO TO 76
   75 FUNC=0.3164/((RE)**(1.0/4.0))
   76 VEL=(8.0*TAU/(FUNC*D))**(1.0/2.0)
      PRINT 11, VEL,V
      DIFF=ABSF((VEL-V)/V)
      IF(DIFF-EP1) 24,24,70
   24 STOP
      END
      SUBROUTINE INTE
      DIMENSION T(401), S(401), PHI(401)
      COMMON NVT,H,GAMMA,DELH,PHI,S,T,SALTH,THETH,G,DELTP,CHDS,CHDT
      DO 10 J=1,NVT
      PHI(J)=0.0
   10 CONTINUE
      IJ=NVT
      PHI(IJ)=THETH
      BIG=GAMMA+1.0
      BIGEX=EXPF(GAMMA)
      BNE=GAMMA-1.0
   20 IJ=IJ-1
      K=IJ+1
      XI=(T(IJ)-(T(K)*BIG))/(GAMMA*BIGEX)
      XJ=((T(IJ)*BNE)+T(K))/GAMMA
      XK=PHI(K)/BIGEX
      PHI(IJ)=XI+XJ+XK
      IF(IJ-1) 30,30,20
   30 RETURN
      END
      SUBROUTINE FINAL
      DIMENSION T(401), S(401), PHI(401)
      COMMON NVT,H,GAMMA,DELH,PHI,S,T,SALTH,THETH,G,DELTP,CHDS,CHDT
      TOTAL=0.0
      I=1
      PRESS=(S(I)-SALTH)*CHDS+(T(I)-PHI(I))*CHDT
   40 I=I+1
      SUMP=((S(I)-SALTH)*CHDS)+((T(I)-PHI(I))*CHDT)
      TOM=DELH*(PRESS+0.5*(SUMP-PRESS))
      PRESS=SUMP
      TOTAL=TOTAL+TOM
      IF(NVT-I) 50,50,40
   50 DELTP=G*TOTAL
      RETURN
      END
      END
```

The computer program is an iteration process for obtaining the velocity of the water in the tube. It was written to solve actual oceanographic cases using the form of the solution given by Groves [2].

Subroutine DELTP is utilized to compute the pressure integral,

$$\Delta p = g\int_{l}^{h}\left\{\left[S(y)-S(h)\right]\frac{\partial\rho}{\partial S}+\left[\theta(y)-\phi(y)\right]\frac{\partial\rho}{\partial\theta}\right\}dy. \qquad \text{(A1)}$$

The subroutine is a trapezoidal integration which assumes a linear relationship between successive points. No greater accuracy is desirable because hydrographic soundings contain salinity and temperature data for relatively few depths, and the linear relationship is generally assumed when plotting the original data.

Subroutine INTE is designed to compute the temperature of the water at each one-meter interval of depth inside the tube using

$$\Phi(y) = \gamma e^{\gamma y}\int_{y}^{h}\theta(\tau)e^{-\gamma\tau}d\tau + \theta(h)e^{-\gamma(h-y)}. \qquad \text{(A2)}$$

Equation (A2) relates water temperature in the tube to depth, but it cannot be solved in this form on the computer since the capacity of the CDC 1604 is exceeded by exponential functions with exponents of magnitude greater than 709. A different form of equation (A2) was necessary for the computer program.

Let $$Z(y) = \gamma e^{\gamma y}\int_{y}^{h}\theta(\tau)e^{-\gamma\tau}d\tau \qquad \text{(A3)}$$

and $$Y(y) = \theta(h)e^{-\gamma(h-y)}. \qquad \text{(A4)}$$

Then $$\Phi(y) = Z(y) + Y(y). \qquad \text{(A5)}$$

In order to get $Z(y-\Delta h)$, we let

$$X(y-\Delta h) = \gamma e^{\gamma(y-\Delta h)}\int_{y-\Delta h}^{y}\theta(\tau)e^{-\gamma\tau}d\tau \qquad \text{(A6)}$$

and then from equations (A3) and (A6)

$$Z(y-\Delta h) = X(y-\Delta h) + e^{-\gamma\Delta h} Z(y). \qquad \text{(A7)}$$

We now assume that $\theta(T)$, where T is a "dummy" variable of integration, varies linearly between consecutive one-meter intervals, and limit the interval, Δh, to one meter.

Then,

$$X(y-\Delta h) = \frac{1}{\gamma}\left\{-e^{-\gamma}\left[(\gamma+1)\theta(y)-\theta(y-\Delta h)\right]+\left[\theta(y)+(\gamma-1)\theta(y-\Delta h)\right]\right\}. \qquad \text{(A8)}$$

Combining equations (A5), (A7), and (A8)

$$\begin{aligned}\Phi(y-\Delta h) = \frac{1}{\gamma}\Big\{&-e^{-\gamma}\left[\theta(y)(\gamma+1)-\theta(y-\Delta h)\right]\\ &+\left[\theta(y)+(\gamma-1)\theta(y-\Delta h)\right]\Big\} + e^{-\gamma}\Phi(y). \qquad \text{(A9)}\end{aligned}$$

Since $\phi(h)=\theta(h)$ is a boundary condition, the first $\phi(y-\Delta h)$ will be calculated for a depth of (h-1) meters. This integration subroutine will function so long as $\gamma < 709$.

APPENDIX B

COMPUTATIONS OF $\partial\rho/\partial\theta$ AND $\partial\rho/\partial S$

The values for the rate of change of density with changing temperature and salinity were assumed constant in the solution of the pressure integral. Additional investigation of the value of these constants was considered important.

Assuming that the effects of pressure are negligible,

$$\rho(S,\theta) = \rho_1 + (S-S_1)\left(\frac{\partial\rho}{\partial S}\right)_1 + (\theta-\theta_1)\left(\frac{\partial\rho}{\partial\theta}\right)_1 + \cdots \text{.} \qquad \text{(B1)}$$

Following Eckart [1],

$$(p+p_o)(\Delta_o - \Delta) = K \qquad \text{(B2)}$$

where

$$\Delta = 10^4\left(1 - \frac{1}{\rho}\right) \qquad \text{(B3)}$$

and

$$p_o = 5890 + 38T + 0.375T^2 + 3S \qquad \text{(B4)}$$

$$K = \left[17.795 + 0.1125T - 0.000745T^2 - (0.0380 + 0.0001T)S\right] \times 10^6 \qquad \text{(B5)}$$

$$\Delta_o = \text{constant} = 3020. \qquad \text{(B6)}$$

Combining equations (B2) and (B3) gives

$$\rho = \left\{1 - \left[10^{-4}\left(\Delta_o - \frac{K}{p+p_o}\right)\right]\right\}^{-1}. \qquad \text{(B7)}$$

Taking the partial derivatives of ρ with respect to θ and S in equation (B7), and evaluating the result at an average temperature, θ=15°C, an average salinity, S=34°/oo, and a zero pressure, gives $\partial\rho/\partial S$ =0.77667kg/m^3·o/ooand $\partial\rho/\partial\theta$ =0.19987kg/m^3·°C.

APPENDIX C

WALL FRICTION CALCULATION

It was considered that the effects of the stress on the water by the wall of the tube might cause significant heating of the water. The work done on the water per unit mass per time is

$$\frac{\tau\, 2\pi r\, dy\, v}{\rho \pi r^2 dy} = \frac{2\tau v}{\rho r} \qquad \left(\frac{\text{joules}}{\text{kg}\cdot\text{sec}}\right). \tag{C1}$$

The work per unit mass is

$$\frac{2\tau v}{\rho r} \cdot \frac{L}{v} = \frac{2\tau L}{\rho r} \qquad \left(\frac{\text{joules}}{\text{kg}}\right). \tag{C2}$$

Dividing by the specific heat, c, and converting joules to calories using 0.2389 calories/joule at 15°C,

$$\frac{2\tau L}{\rho c r}(0.2389) = \Delta T \qquad (^\circ\text{C}). \tag{C3}$$

Applying the data used in the solution of case #1, the temperature change of the water in the tube due to wall friction is 0.00025°C for a velocity of 17.32 cps and a pipe length of 600 meters. This change in temperature is negligible.

APPENDIX D

COEFFICIENT OF VERTICAL HEAT CONDUCTION FOR TURBULENT FLOW

The value for the coefficient of vertical heat conduction for turbulent flow, K, was investigated. From Schlichting, [6], the coefficient of eddy conductivity of heat, A_q, is at least as large as the coefficient of eddy viscosity, A_τ, over the entire pipe. We assume the turbulence to be approximately isotropic, so that turbulent transfer along the pipe is the same order of magnitude as that in any cross section.

A good estimate of the effective value of A_q can then be obtained by calculating an average of A_τ over a cross section. This is easily done using mixing length theory. The result is

$$\overline{A_q} = \overline{A_\tau} = \frac{\rho}{\pi r^2}\int_0^r k^2 u_* z \, 2\pi (r-z)\, dz = \rho \frac{k^2 u_* r}{3} \qquad \text{(D1)}$$

where r is the interior radius of the tube, z is measured inward from the wall, u_* is the "shear velocity" (see Schlichting), and $k=0.4$.

Then,

$$K = \overline{A_q}\, c_p \, . \qquad \text{(D2)}$$

This has a typical value of 5.0 cal./m·°C·sec. for the rates of flow and tube size considered.

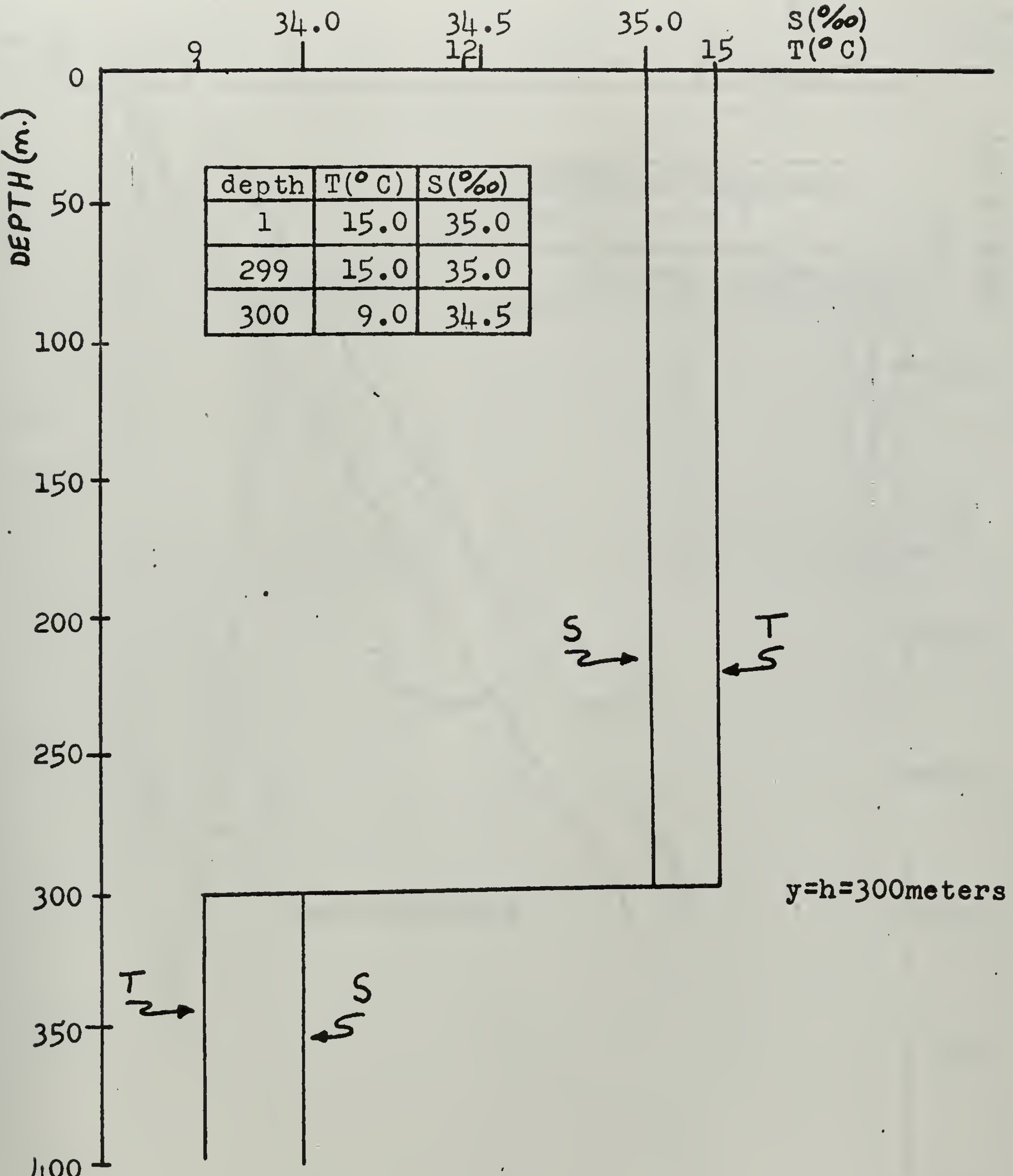
APPENDIX E

"FIXED PARAMETER" VALUES

The expression "fixed parameters" refers to values for kinematic viscosity of sea water (ν), conductivity of tube material (n), density of sea water (ρ), specific heat of sea water (c), acceleration of gravity (g), rate of change of density with changing salinity ($\partial\rho/\partial S$) and rate of change of density with changing temperature ($\partial\rho/\partial\theta$). Two sets of values for the "fixed parameters" have been used. One set contains rounded-off values for hand calculations, and the other set contains more precise values for the computer program. One could then determine the effect on rate of flow of changes in the values of the "fixed parameters."

	APPROXIMATE	PRECISE	UNITS
ν	10^{-6}	$1.22\text{X}10^{-6}$	m^2/sec
m (copper)	10^2	10^2	cal/m·°C.sec
ρ	10^3	$1.026\text{X}10^3$	kgm/m^3
c	10^3	$9.33\text{X}10^2$	cal/kgm·°C
g	10	9.8	m/sec^2
$\partial\rho/\partial S$	0.7	0.77667	kgm/m^3
$\partial\rho/\partial\theta$	-0.2	-0.19987	kgm/m^3·°C

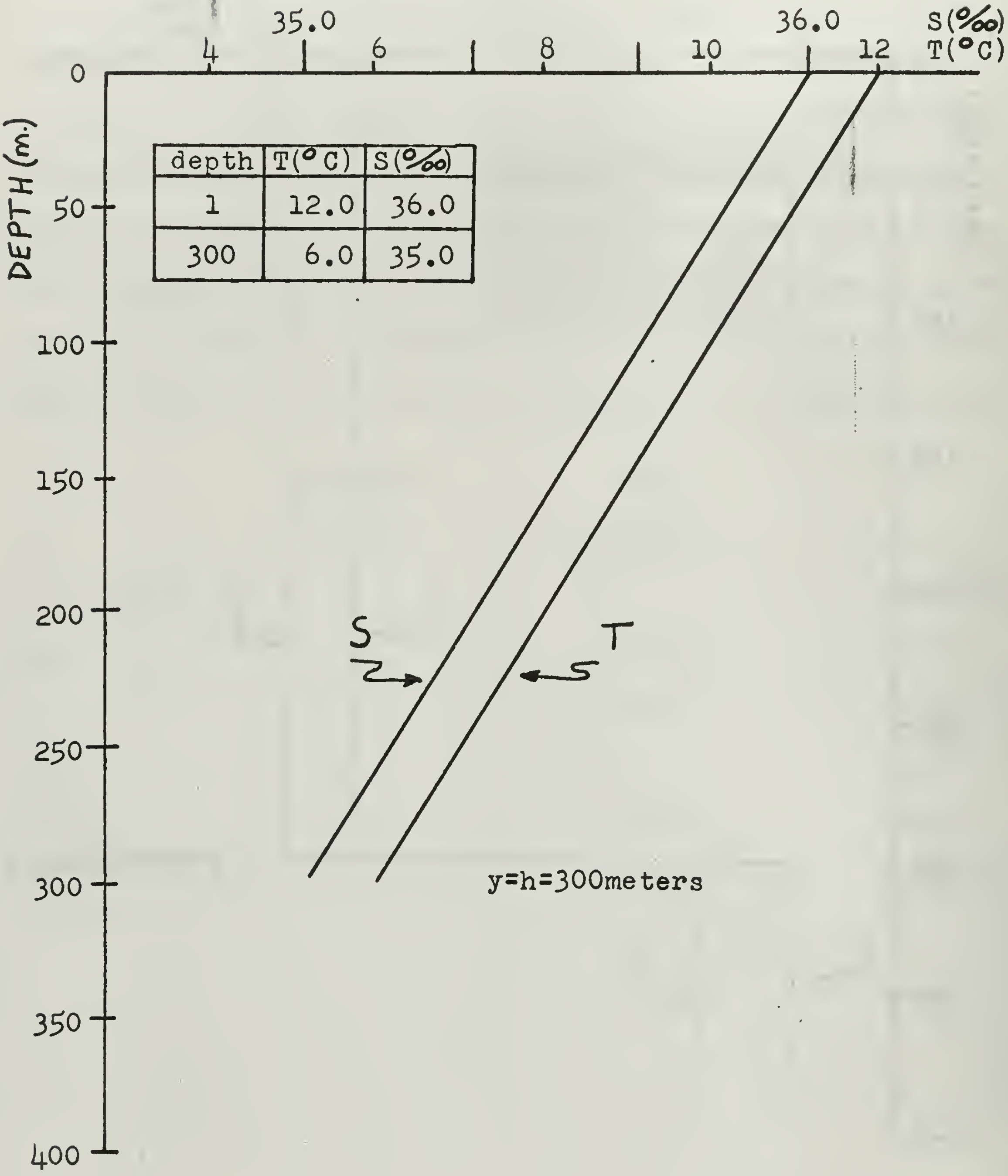
INITIAL TEMPERATURE-SALINITY PROFILES CONSTRUCTED TO APPROXIMATE GROVES' DATA



depth	T(°C)	S(‰)
1	15.0	35.0
299	15.0	35.0
300	9.0	34.5

Figure 1.

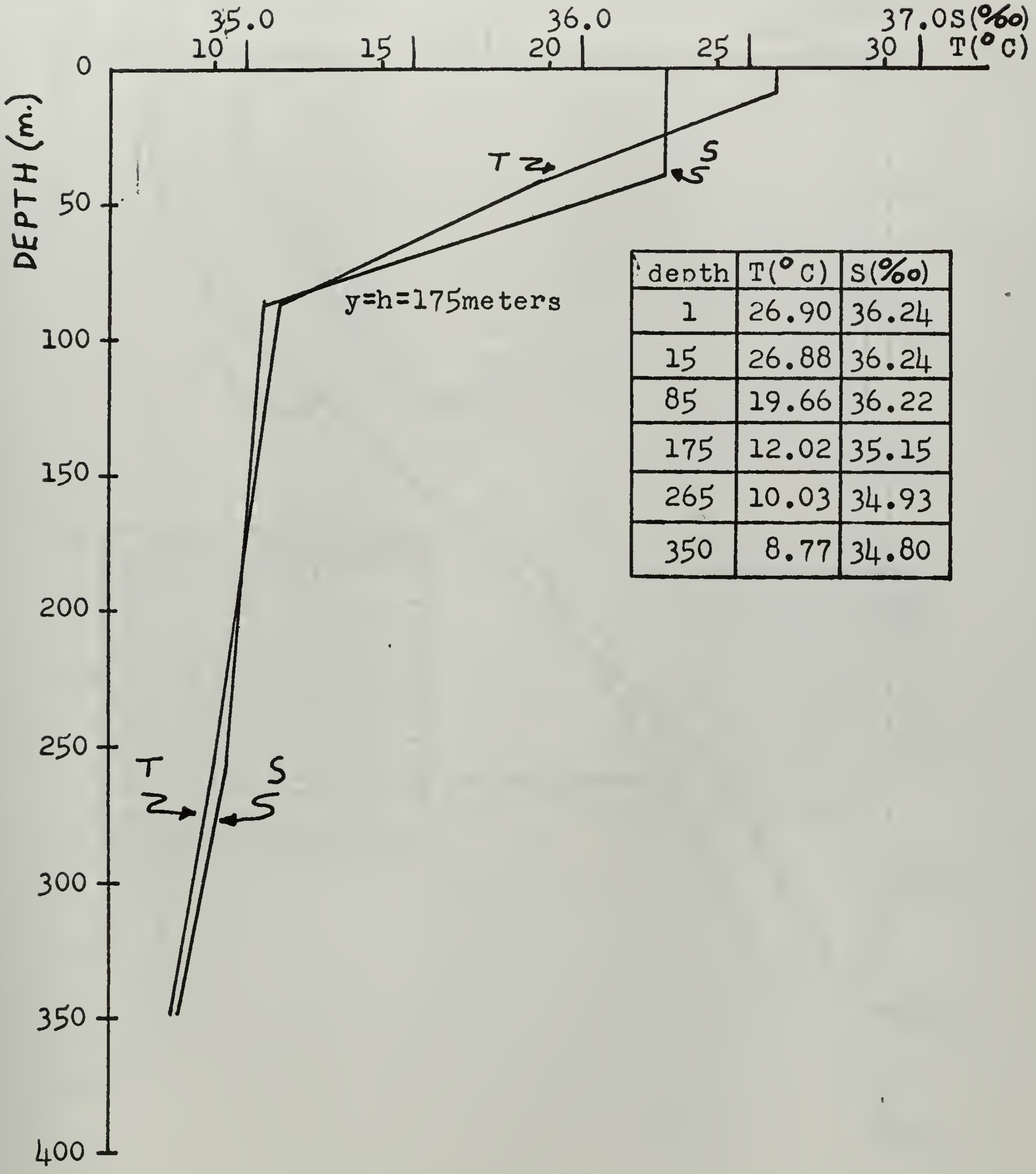
LINEAR TEMPERATURE-SALINITY PROFILES



depth	T(°C)	S(‰)
1	12.0	36.0
300	6.0	35.0

Figure 2.

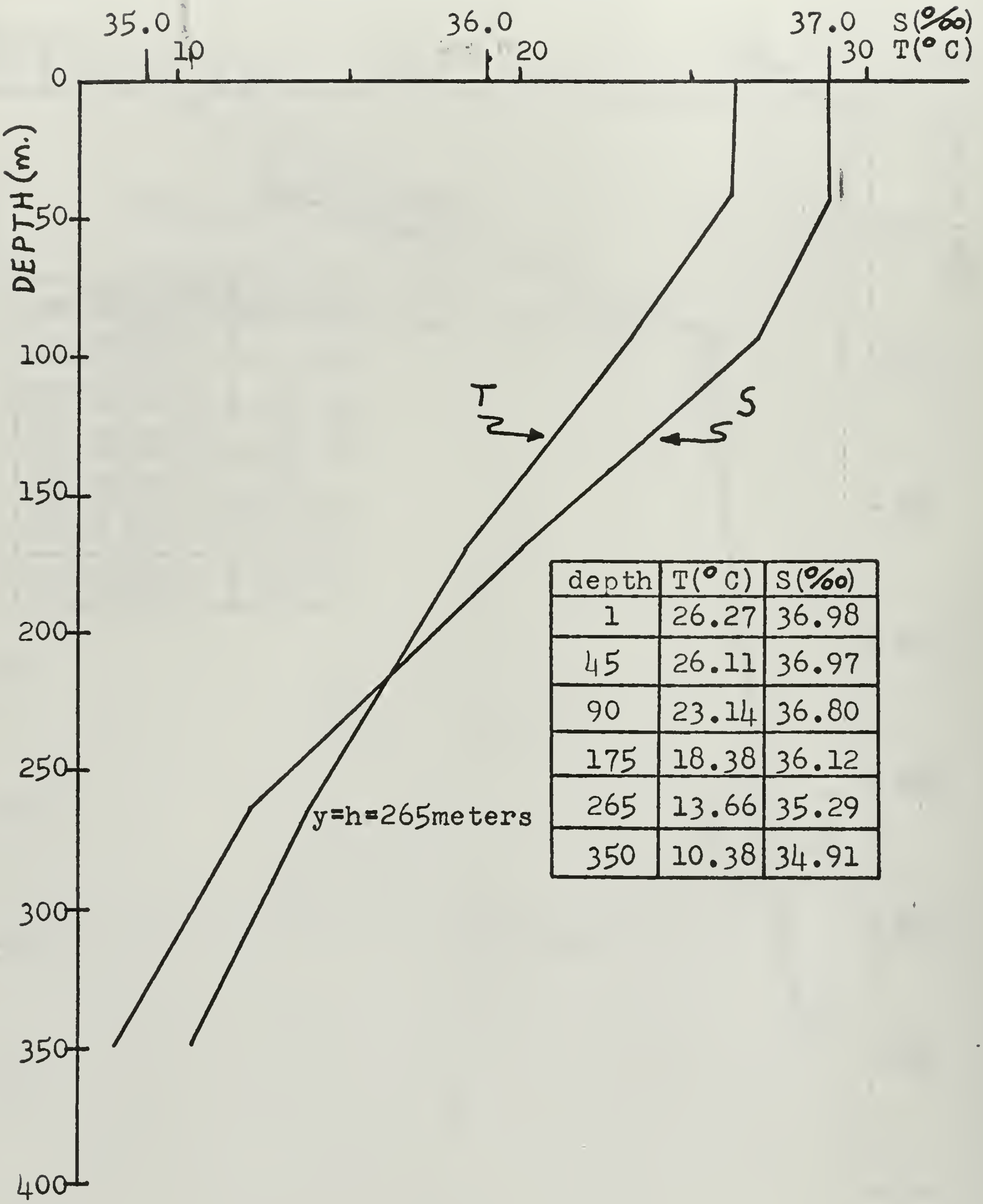
ACTUAL TEMPERATURE-SALINITY PROFILES



depth	T(°C)	S(‰)
1	26.90	36.24
15	26.88	36.24
85	19.66	36.22
175	12.02	35.15
265	10.03	34.93
350	8.77	34.80

Figure 3.

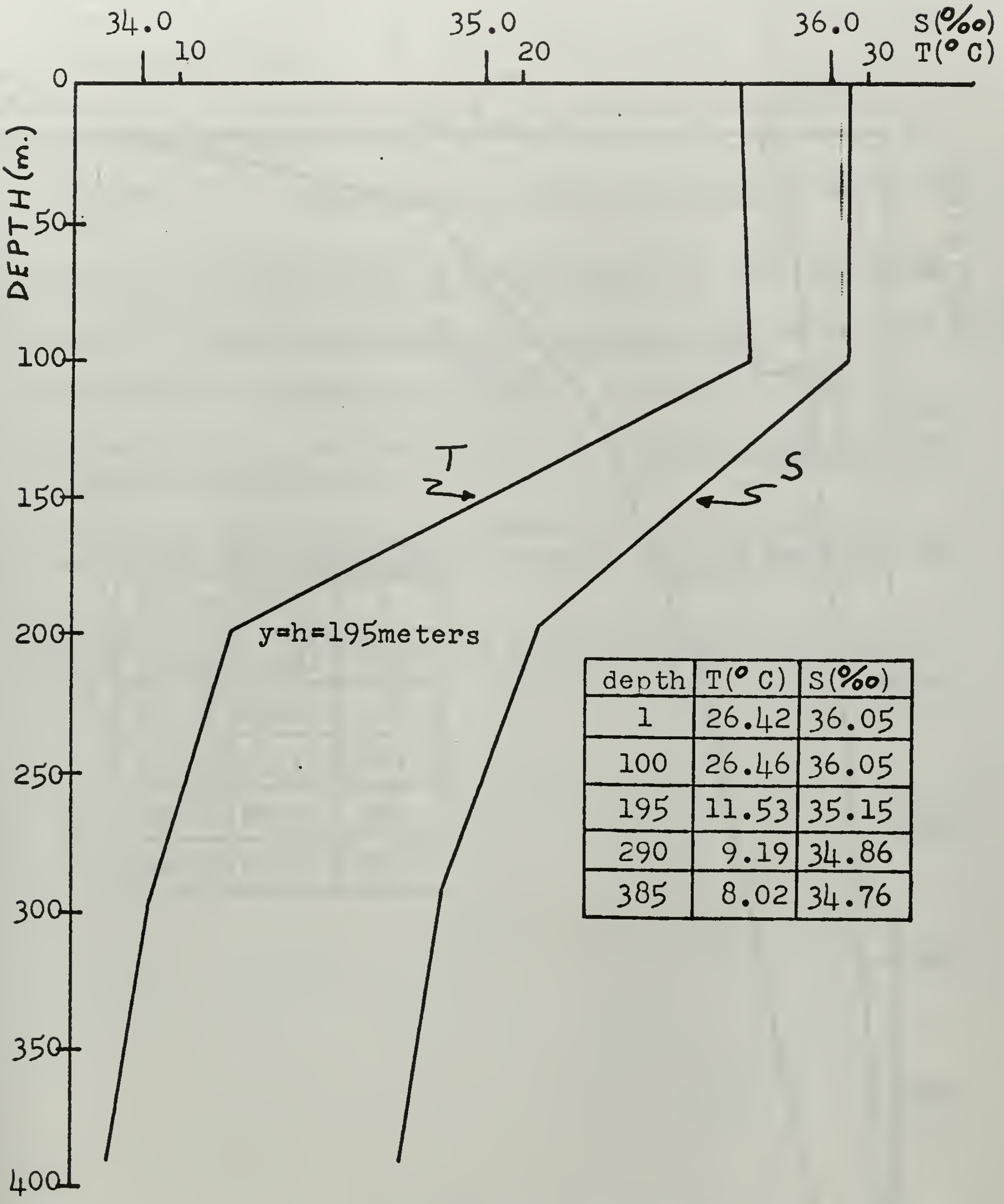
ACTUAL TEMPERATURE-SALINITY PROFILES



depth	T(° C)	S(‰)
1	26.27	36.98
45	26.11	36.97
90	23.14	36.80
175	18.38	36.12
265	13.66	35.29
350	10.38	34.91

Figure 4.

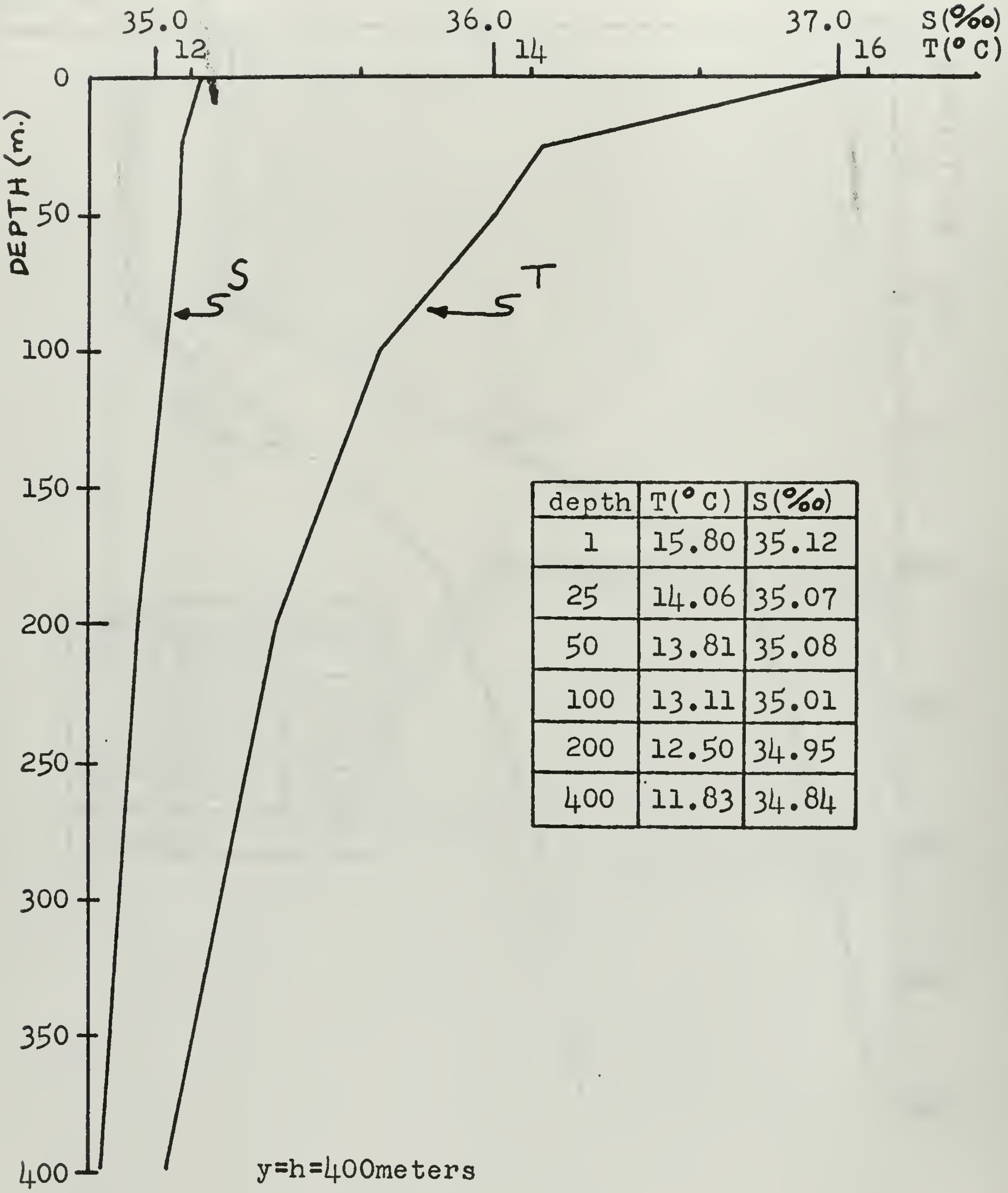
ACTUAL TEMPERATURE-SALINITY PROFILES



depth	T(° C)	S(‰)
1	26.42	36.05
100	26.46	36.05
195	11.53	35.15
290	9.19	34.86
385	8.02	34.76

Figure 5.

ACTUAL TEMPERATURE-SALINITY PROFILES



depth	T(°C)	S(‰)
1	15.80	35.12
25	14.06	35.07
50	13.81	35.08
100	13.11	35.01
200	12.50	34.95
400	11.83	34.84

Figure 6.

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13. ABSTRACT

The "perpetual salt fountain" can occur in an area of the ocean where the water is vertically stable. Warm, saline water overlies cold, relatively fresh water. A tube is placed in the water from just beneath the ocean surface to just beneath the halocline. Water is pumped up the tube to initiate the flow. As the water in the tube is warmed, it becomes less dense than (and thus buoyant with respect to)the surrounding water, and the flow will continue after pumping is stopped. The effect of the vertical conduction of heat by turbulent mixing on the velocity of flow in the "perpetual salt fountain" is investigated. Temperature and salinity profiles with depth from hydrographic soundings are considered to determine flow velocities in the ocean. The calculations show that vertical heat conduction has a negligible effect on the rate of flow in the presence of horizontal heat conduction, the salinity-depth profile is the primary factor influencing the rate of flow, and the temperature-depth profile has a negligible effect on flow velocity.

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